APPENDIX 1 - The funding application (to be filled out in English)

B. The project leader

B1. Curriculum Vitae of the project leader[1] (*maximum 2 pages)*

Name: Budaca Radu, **Date of Birth**: 01.02.1984, **Email**: rbudaca@theory.nipne.com

Position: Scientific Researcher III, Department of Theoretical Physics, "*Horia Hulubei*" National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania.

Professional address: "*Horia Hulubei*" National Institute of Physics and Nuclear Engineering (IFIN-HH), P.O. Box MG-6, 077125- Magurele, Romania, Fax +(4021) 457.44.40

Education: 2012 - Ph.D. in Theoretical Physics, University of Bucharest. Thesis: "*Contributions to the phenomenological and microscopic description of nuclear and atomic systems*". Supervisor: Prof. Em. Dr. Apolodor Aristotel Raduta. **2008 - Master Degree** in Computational Physics, Econophysics, Theoretical Physics and the Physics of Condensed Matter, Polymer Physics, University of Bucharest. Thesis: "*Semiclassical description of triaxial rigid rotor*". Supervisor: Prof. Em. Dr. Apolodor Aristotel Raduta. **2006 - Bachelor Degree** in Physics, University of Bucharest. Thesis: "*Boson representations of angular momentum*". Supervisor: Prof. Em. Dr. Apolodor Aristotel Raduta.

Professional experience: **2013 - Present** Scientific Researcher III, **2012 - 2013** Scientific Researcher, **2007 - 2012** Research Assistant in the Department of Theoretical Physics, "*Horia Hulubei*" National Institute of Physics and Nuclear Engineering.

Training: 2014 - International School of Nuclear Physics, 36-th Course "*Nuclei in the Laboratory and in the Cosmos*", Erice-Sicily, Italy. **2012** - International Summer School for Advanced Studies "*Dynamics of open nuclear systems*", Predeal, Romania. **2011** - Helmholtz International Summer School "*Nuclear Theory and Astrophysical Applications*", Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Russia. **2011** - Doctoral training program "*Neutrinos in Nuclear-, Particle- and Astrophysics*", European Center for Theoretical Studies in Nuclear Physics and Related Areas (ECT*), Trento, Italy. **2006** - International Summer School in Nuclear Physics "*Collective motion and Phase Transitions in Nuclear Systems*", Predeal, Romania. **2005** - DAAD Summer School "*Trends in Contemporary Optics*", Sinaia, Romania.

Visits longer than 30 days: 2011 (over 2 months) Doctoral training program ECT*, Trento, Italy. 2008 (2 months), 2009 (2 months), 2010 (one month): Visiting student, working on a DFG program at

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[1] [2] Sections B1 and B3 of the application will be published on UEFISCDI website. These will be uploaded in the online platform for submission, both as two separate pdf. files as well as part of the funding application.

Institute for Theoretical Physics with Prof. Amand Faessler, University of Tuebingen, Germany. **Awards, Scholarships and Fellowships**: **2011** - Doctoral Training Program Fellowship, ECT*, Trento, Italy. **2009** – "*Horia Hulubei*" Award, Romanian Academy. For the set of papers: "*Manifestation of the atomic clusters as many body systems of small and medium particles*". **2007 and 2008** - "*Serban Titeica*" Prize, "Horia Hulubei" National Institute of Physics and Nuclear Engineering. Best scientific contribution of young scientist under 35 years. **2008** - One of the three Ph.D. Scholarship for students from Republic of Moldova, University of Bucharest. **Invited Talks**: **2015** - International Workshop "*Shapes and Dynamics of Atomic Nuclei: Contemporary Aspects*" (SDANCA-15), Sofia, Bulgaria; **2013** - 13-rd International Balkan Workshop on Applied Physics and Material Science, Constanta, Romania. **Contribution Talks**: SSNET Workshop "*Shapes and Symmetries in Nuclei: from Experiment to Theory*", Gif-sur-Yvette, France (2016); TIM 15-16 Physics Conference, Timisoara, Romania (2016); European Nuclear Physics Conference "*The Future of Nuclear Physics, Today!*", Groningen, Netherlands (2015); TIM14 Physics Conference "*Physics without frontiers*", Timisoara, Romania (2014); 2-nd European Nuclear Physics Conference, Bucharest, Romania (2012); International Summer School for Advanced Studies "*Dynamics of open nuclear systems*", Predeal, Romania (2012); Helmholtz International Summer School "*Nuclear Theory and Astrophysical Applications*", Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research (JINR), Dubna, Russia (2011). **Poster presentations**: SSNET Workshop "*Shapes and Symmetries in Nuclei: from Experiment to Theory*", Gif-sur-Yvette, France (2016); 26-th International Nuclear Physics Conference INPC2016, Adelaide, Australia (2016); 28-th EPS Nuclear Physics Divisional Conference "*Nuclear Physics in Astrophysics VII*", York, UK (2015); 25-th International Nuclear Physics Conference INPC2013, Firenze, Italy, (2013). **Publications**: 24 Peer Reviewed Publications in prestigious nuclear, atomic and multidisciplinary physics journals such as Phys. Lett. B (3), Ann. Phys. (3), Phys. Rev. C (4) and A (1), J. Phys. G: Nucl. Part. Phys. (4), Eur. Phys. J. A (3), Nucl. Phys. A (1), Mod. Phys. Lett. A (1), Int. J. Mod. Phys. B (1), J. Supercond. Nov. Magn. (1) and Rom. J. Phys. (2) with a total of 62 citations. 12 publications are as principal author. Authored and coauthored 11 Proceedings papers. **Reviewer for**: The European Physical Journal A (2015), Nuclear Physics A (2x2016), International Journal of Modern Physics E (3x2016), Acta Physica Polonica A (2016), Romanian Journal of Physics (2013). **Languages**: Native in Romanian. Fluent in both written and spoken Russian and English. **Computer and Programming Skills**: Experienced on user level with Microsoft Windows and with the most flavors of Linux. Advanced skills on numerical algorithms in Fortran and symbolic calculus in Mathematica and Maple. Generic knowledge of C++ and Pascal. Experienced with Origin, Mathematica, Maple and Grace. Excellent skills in Latex, Microsoft Office and Open Office Suites.

Member in National Research Projects: "*Excellency research in the fields of nuclear structure and double β decay*", ID number - Idei/33/28.09.07 / 28.08.07, Director A. A. Raduta, Position - Ph.D. Student; "*Topical research on nuclear structure, phase transitions and double β decay*", ID number - PN-II-ID-PCE-2011-3-0042, Director A. A. Raduta, Position - PostDoc.

B2. Significant and representative scientific achievements *(maximum 2 pages)*

The most important contributions of the Project Leader (PL) to the field of theoretical atomic and nuclear physics are:

1) A new boson representation for the SU(2) algebra of angular momentum operators. This was obtained in connection to a semiclassical treatment of the triaxial rigid rotor through a time dependent variational principle, using an angular momentum coherent state as a variational function [Phys. Rev. C 76 (2007) 064309]. **2) A compact formula for ground band energies of even-even nuclei**. Using a time dependent variational principle for the semiclassical description of a second order boson Hamiltonian, PL obtained a two-parameter expression [J. Phys. G 37 (2010) 085108] resembling the Holmberg-Lipas formula. The superior predictive power of the proposed formula has been proved through extensive numerical calculations on 92 nuclei exhibiting various spectral properties. **3) Theoretical prediction of the volume plasmon resonance in atomic clusters**. Speculating the similarity of nuclear systems with atomic clusters, some properties of small and medium sodium clusters related to collective dipole excitations were studied by the PL and collaborators within the random phase approximation using a projected spherical single-particle basis [Phys. Rev. A 79 (2009) 023202]. The major result of this study was the identification of the ultraviolet shifted peak in the photoabsorbtion cross section associated to the volume plasmon resonance which is a consequence of the finite size effects of the nano clusters in their interaction with light. The result was shortly validated experimentally with a full citation of PL's work. A byproduct of this study is the unique description of the static electric polarizability, which is fully microscopical. **4) Modification of the Thomas-Reiche-Kuhn sum rule for the Schiff dipole transition operator** and its application to clusters of Na atoms [Int. J. Mod. Phys. B 25 (2011) 467; J. Supercond. Nov. Magn. 24 (2011) 645]. **5) A semi-microscopic description of the simple and double backbending phenomena in even-even nuclei**. Based on the competition of single-particle and collective degrees of freedom, a detailed analysis of the neutron pair breaking, as well as of the alignment mechanism of individual angular momenta to the core angular momentum was realized, which pointed out several new features of the backbending phenomenon [Phys. Rev. C 84 (2011) 044323]. Later, PL extended the model to the more rare phenomenon of double backbending [J. Phys. G 40 (2013) 025109], where the second event is due to an additional proton broken pair. **6) Various contributions to the Coherent State Model (CSM)**. PL in collaboration generated compact formulas for the vibrational and asymptotic regimes of deformation in CSM [Ann. Phys. 327 (2012) 671] and established the relation between the Nilsson basis and the projected spherical single-particle basis having as a component the collective CSM wave-function [Ann. Phys. 347 (2014) 141]. **7) An approximate solution for the γ-rigid Bohr Hamiltonian with anharmonic potentials**. An analytical expression for the energy spectrum of the ground and β excited bands was obtained by PL in the axially symmetric γ-rigid regime of the Bohr Hamiltonian with a harmonic oscillator potential in the β shape variable amended with a quartic [Eur. Phys. J. A 50 (2014) 87] or sextic [Phys. Lett. B 739 (2014) 56] anharmonic term. **8) The first description of the shape phase transition in the Bohr-Mottelson Model**. The PL studied for the first time the shape phase transition in axial [J. Phys. G 42 (2015) 105106] and triaxial [Phys. Rev. C 91 (2015) 014306; Ann. Phys. (NY) 375 (2016) 65] nuclei by means of a quasi-exactly solvable version of a sextic potential in the Bohr-Mottelson Model. This property offers the possibility to study the shape evolution within an isotopic chain and identify possible critical point nuclei in a unified model. The resulted papers became reference points of the topic with a high rate of citation. **9) The introduction of the energy dependent collective phenomena**. The PL used for the first time the theory of energy dependent potentials in the Bohr Hamiltonian to obtain collective spectra with new spectral characteristics which bring an important addition to the exactly solvable collective solutions. Moreover, in special conditions PL obtained a new parameter free model called Stiffening Spherical Vibrator [Phys. Lett. B 751 (2015) 39]. **10) Proposal of the γ-rigid/stable shape phase mixing mechanism**. An alternative approach to the collective phenomena was obtained through a coherent interplay of γ-rigid and γ-stable collective conditions described in distinct shape phase spaces. This allowed the study of various critical point solutions as function of a rigidity measure of the nuclear surface [J. Phys. G 42 (2015) 085103, Eur. Phys. J. A 51 (2015) 126]. **11) A new critical point partial dynamical symmetry**. Using the shape phase mixing mechanism to study the critical point of the shape phase transition between spherical and axially symmetric nuclei, PL found that in the equally mixed case, the model called X(4) exhibits properties of the Euclidean symmetry in four dimensions [Phys. Lett. B 759 (2016) 349, Phys. Rev. C 94 (2016) 054306]. **12) A new empirical formula for the α decay half-lives of superheavy nuclei**. The formula was obtained by modifying the Brown formula and further used for prediction of the decay properties for 125 unknown superheavy α emitters [Nucl. Phys. A 951 (2016) 60]. **13) The first fully geometrical description of shape coexistence phenomenon in nuclei**. Solving an energy dependent Coulomb-like potential in the γ-unstable regime of Bohr-Mottelson model, PL obtained a collective spectrum with an extremely low 0+ excited state which is considered as the major signature of the shape coexistence. [Eur. Phys. J. A 52 (2016) 314].

B3. Defining elements of the outstanding scientific achievements of the project leader[2] *(maximum 3 pages)*

1. Articles

   1.1. **R. Budaca**, "*Bohr Hamiltonian with an energy dependent γ-unstable Coulomb-like potential*", The European Physical Journal A 52, 314 (2016). Indexed in Web of Science.

   1.2. **R. Budaca**, P. Buganu, M. Chabab, A. Lahbas, and M. Oulne, "*Extended study on a quasi-exact solution of the Bohr Hamiltonian*", Annals of Physics (New York) 375, 65 (2016). Indexed in Web of Science.

   1.3. **R. Budaca** and A. I. Budaca, "*Emergence of Euclidean dynamical symmetry as a consequence of shape phase mixing*", Physics Letters B 759, 349 (2016). Indexed in Web of Science.

   1.4. **R. Budaca**, "*Spherical vibrator model with an energy increasing stiffness*", Physics Letters B 751, 39 (2015). Indexed Web of Science. (2 citations)

   1.5. **R. Budaca**, "*Harmonic oscillator potential with a sextic anharmonicity in the prolate γ-rigid collective geometrical model*", Physics Letters B 739, 56 (2014). Indexed in Web of Science. (3 citations)

2. Books/ chapters (including monographs):

   2.1. P. Buganu and **R. Budaca**, "Z(4)-Sextic: A γ-rigid solution of the Bohr Hamiltonian with sextic oscillator potential for β and γ=30°" in "Nuclear Structure and Dynamics '15", Book Series: AIP Conference Proceedings, Volume 1681, Article Number 040014, Year 2015, ISBN 978-0-7354-1328-3.

   2.2. **R. Budaca**, "Harmonic oscillator potential with a quartic anharmonicity in the prolate γ-rigid collective geometrical model" in "TIM14 Physics Conference: Physics without Frontiers", Book Series: AIP Conference Proceedings, Volume 1694, Article Number 020010, Year 2015, ISBN 978-0-7354-1341-2.

   2.3. P. Buganu and **R. Budaca**, "Anharmonic vibrations around a triaxial nuclear deformation frozen to γ=30°" in "TIM14 Physics Conference: Physics without Frontiers", Book Series: AIP Conference Proceedings, Volume 1694, Article Number 020006, Year 2015, ISBN 978-0-7354-1341-2.

   2.4. **R. Budaca** and A. A. Raduta, "Semi-microscopic description of the proton- and neutron-induced backbending phenomena in some deformed even-even rare earth nuclei" in "INPC 2013 -

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International Nuclear Physics Conference, Vol. 1", Book Series: EPJ Web of Conferences, Volume 66, Article Number 02017, Year 2014, ISBN 978-2-7598-1175-5.

3. Research projects:

Note[3]:

To have the chance for funding recommendation, the project leader must demonstrate an internationally visibility average or above average activity at paragraphs 1 or 2, in Section B3. Average or above average activity in the international community:

- ***For Science and Social Sciences,*** **the activity with an average or above average visibility is highly defined by the existence of at least two publication as main author (document type: „article" or „review") published in journals situated in the "top 50% of all journals published in the research field of the project proposal". The reference list with journals belonging to top 50%, red and yellow areas, is achieved according to the scores of influence (AIS) calculated by Thomson Reuters in Journal Citation Reports 2015 published by Thomson Reuters in June 2016:**
  http://uefiscdi.gov.ro/userfiles/file/PNCDI%20III/P1_Resurse%20Umane/PRECISI_2016/PRECISI_lista%20AIS%202016.pdf
- ***For Arts and Humanities,*** **the activity with an average or above average visibility is highly defined by the existence of at least two publication as main author (document type: „article" or „review") published in journals indexed in Arts and Humanities Citation Index:**
  http://ip-science.thomsonreuters.com/cgi-bin/jrnlst/jlresults.cgi?PC=H
- **Complementary routes for Social Sciences and Humanities:**
  - ✓ **ERIH Plus (Int categories 1 and 2 of the evaluation from 2011):** **http://uefiscdi.gov.ro/userfiles/file/CENAPOSS/clasificare_reviste_ERIH.pdf**
  - ✓ **Specific publications for the application field, published in prestigious international publishing houses, for social sciences, arts and humanities fields, according to the following lists:** **http://uefiscdi.gov.ro/userfiles/file/CENAPOSS/Edituri%20prestigiu%20international%20stiinte%20sociale.pdf**

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[3] Please, keep the note! It is not quantified by a number of pages and it is useful for evaluators.

http://uefiscdi.gov.ro/userfiles/file/CENAPOSS/Edituri%20prestigiu%20international_Arte%20&%20Stiinte%20Umaniste.pdf

The main authorship is expressed depending on the field, the project leader having the responsibility to present it correctly, in relation to the standards from the field, the authorship following to be also analyzed by the panel and by the evaluators from the field; where the academic norm is to list the authors alphabetically, there will be presented the publications as co-author (with the argumentation of those academic practices).
The lack of average or above average activities to the paragraphs 1 or 2 reduces the chance for the project proposal to be financed.

C. Research project description *(maximum 10 pages and maximum 1 page bibliography)*

C1. Issues

The main concern of the project is to investigate the properties of atomic nuclei which stem from deformation. Despite the huge variety of their spectral properties, these complex objects display astonishing simplicities and regularities. Thus, the main questions which are to be answered by the project's results are: What are the symmetries underlying these simple patterns and what is their origin? In order to answer these fundamental questions of the nuclear physics, one must start from the premise that deformation is essentially a macroscopic feature. This in turn calls for a many-body perspective on the nucleus as a whole with its own shapes, excitation modes, symmetries, many-body quantum numbers, and selection rules. The low lying energy levels of deformed nuclei are usually described by means of intrinsic states of rotational, vibrational or even non-collective nature with superposed rotational sequences on top of each such band head states. The collective intrinsic excitations are in general rotations and quadrupole vibrations that preserve (β mode) and break (γ mode) axial symmetry. Spherical nuclei usually found near closed shells have only few valence nucleons of each type, whose two-particle two-hole excitations combine coherently into multi-phonon states of the nuclear surface vibration. While the collective motion of deformed nuclei is a complex coupling between rotation and vibration. The deformation of nuclear shape comes as a consequence of the single-particle configuration mixing appearing in large valence spaces, which facilitates stable quadrupole deformation minima for the total ground state energy. As structure develops between these limiting cases, there must be nuclei with intermediate or transitional character that are neither spherical nor well deformed. The hypotheses about their nature oscillate between thermodynamic-like critical points of shape phase transitions (SPT) [1] and coexistence of multiple mixed or decoupled shapes [2]. The SPT in this case is a quantum one and it concerns the equilibrium shape and structure of the ground and low-lying states. The critical point description of transitional nuclei is actually in agreement with the coexistence of highly mixed shapes, because their corresponding potential energy surfaces exhibit multiple minima in the deformation space with shallow barriers. Although initially thought as an exotic possibility, shape coexistence is presently considered to occur in most nuclei heavy enough to manifest collectivity. Its experimental manifestation is however hindered by the various degrees of shape mixing and its relevance to the low energy spectrum. Despite the great progress in microscopic many-body theories of nuclear systems, it is still not possible to directly describe the collective phenomena by employing single-particle degrees of freedom. The adiabatic time-dependent Hartree–Fock–Bogolyubov theory can provide such a description, but it leads to a classical collective Hamiltonian, whose quantisation is ambiguous. On the other hand, the random phase approximation is not able to explain a large amplitude collective motion because it assumes harmonicity of the excitations. The most successful method in relating single-particle and collective degrees of freedom is the Generator Coordinate Method which can yield a collective Hamiltonian only in its Gaussian overlap approximation. The microscopic input for this method includes phenomenological mean field models, self-consistent approaches with effective nucleon–nucleon forces and the relativistic mean field theory. The notion of shape in these microscopic theories is however abstract, being related to the geometry of the mean field or the symmetry of the adopted interaction which must not be confounded with the actual nuclear shape. The most advanced current theories of this kind are in general massively computational. Therefore, semiclassical approaches and phenomenological models based on the geometry and symmetry of the nuclear surface dynamics become more viable alternatives. The overall success of such models highlights again the remarkable fact that, although nuclei are composed of up to hundreds of protons and neutrons, their expected high complexity is not observed. Instead one sees amazingly simple regularities which are undeniably ascribed to the emergence of collectivity in many-body systems. The associated bulk properties are traditionally described by means of collective models. In particular, the Bohr-Mottelson model [3,4] (BMM) of nuclear surface oscillations provides an intuitive phenomenology as well as a fully quantum description of the collective motion. Its algebraic structure allows the realisation of the three dynamical symmetries (DS): U(5) [3], O(6) [5] and SU(3) [4] which constitute the subgroup branches of the U(6) group underlying the Interacting Boson Model (IBM) [6]. The correspondence of the BMM results with IBM which is actually a drastic truncation of the shell model, bridges the single-particle degrees of freedom with the purely geometrical collective variables. BMM acts in a five dimensional shape phase space composed of the β and γ shape variables associated to the deviations from spherical and respectively axial symmetry, and three Euler angles describing the rotation of the nucleus. The associated Bohr Hamiltonian (BH) is a sum of a kinetic operator defined by a mass bi-tensor within a Pauli-Podolski quantization prescription [7] and a potential energy depending on the shape variables. The general BH with variable mass parameters cannot be solved neither analytically nor numerically [8] even for simple potentials. It has however exactly solvable limits [9,10] and computationally tractable instances [11,12]. In both cases, the complexity of the potential is limited to having a single minimum in the space of β and γ variables. This is one of the major drawbacks of the model, because it cannot be effectively applied to transitional nuclei. A loophole in this sense was found by Iachello, who introduced particular BH solutions with an infinite square well potential (ISW) [13], which shortly became reference pictures for critical collective nuclear phenomena [14]. As a consequence of the exact solvability of BH with ISW, the corresponding solutions were found to be closely related to the Euclidean DS [15] which is a semi-direct sum between a rotation group and a translation group. More specifically, the Euclidean DS is exactly realized in the critical point of the shape phase transition between spherical vibrator and the γ-soft rotor limits, and partially for critical points of the shape phase transitions between spherical vibrator and axial rigid rotor and respectively between prolate and oblate shapes [13,16]. The five-dimensional Euclidean DS also emerges at the triple point of the shape phase diagram of the IMB [17]. It is then natural to assume that the critical point of SPT, which is associated with a potential having double minima must have an algebraic structure incorporating the Euclidean DS. The project undertakes the realization of a theoretical program which would be able to solve the BH for potentials with multiple minima, bridging thus the isolated success of BH with various shapes of the potential. The problem reduces to finding the right diagonalization basis, which will assure fast convergence for the energy levels. The lowest order potential in the β variable which has two minima is the sextic oscillator potential. It also allows single spherical and deformed minima, recommending it as the simplest general potential covering all deformation regimes, including the transitional one where shape coexistence resides. The critical point of a SPT between spherical and deformed nuclei is conditioned by a potential with spherical and deformed minima of the same depth. Otherwise the nucleus will favor the deformation corresponding to the lowest minimum and will no longer be critical but merely exhibiting a kind of shape coexistence [14]. Depending on the relative height of the two minima, the shape coexistence can reveal itself only for higher excited states or could be quite imperceptible. The magnitude of the separating potential barrier dictates the degree of the mixing between the two coexisting deformation structures. All these aspects can be treated in a unified manner by considering a general sextic potential in the BH. Shape coexistence can take also other forms for which alternative approaches must be sought [18]. The evolution of deformation with energy is one such case. Recently, PL used for the first time the theory of non-local potentials within the BH to treat such exotic collective excitations [19]. Thus, depending on the potential, the associated nuclear shape can become stiffer or softer with increasing energy, creating proper collective condition for state depending shape coexistence. Due to mathematical constraints, already obtained solutions are parameter-free and therefore serve only as reference points. In order to use the full capacity of this theory, PL will apply the same treatment to other more pliable exactly solvable potentials. The aim of such an endeavor is to justify the adequacy of non-local potentials to the shape coexistence by investigating the evolution of its various signatures (low excited 0+ states, enhanced monopole transitions, parallel bands, etc.) as a function of potential parameters, and to provide a fully geometrical alternative for the qualitative and quantitative description of this phenomenon. One of the mentioned aims of the project is to explore symmetries governing the evolution of collective behavior. Consequently, a special attention will be devoted to the study of the algebraic structure of the adopted diagonalization basis and its symmetry obeying aspects. The presence of a dynamical symmetry is directly related to the exact solvability of the associated Hamiltonian. The search for these symmetries in the proposed solutions based on energy dependent potentials is another target of the project. Finally, a throughout comparison of the all developed models to the results of other algebraic approaches such as IBM and Symplectic Shell Model [20] which have microscopic foundations is also on the project's agenda.

C2. Objectives

The objectives of the project are based on two premises: a) The problems treated in the project are at the forefront of the respective field; b) The project will assure the continuity and further development of the earlier valuable research and original ideas of the PL's team. There are two major objectives:

1. Construction of an algebraic collective model based on a generalised method for solving BH with a most general potential in shape variables allowing multiple deformation minima. The objective have intermediate work packages (WP) with their own stand alone importance. As the lowest order BH symmetry obeying potential is the sextic oscillator, the study will be centered on its solution within different regimes of the BMM. For simplicity reasons, the program of the present objective will commence by diagonalizing a separated β equation with a sextic potential obtained from an adiabatic decoupling of β and γ shape fluctuations (**WP 1.1**). This separable instance of the BH constitute the perfect ground for searching and testing the structure of the most fitting diagonalization basis and is also plentiful in experimental realizations. By diagonalizing the BH with a sextic potential in the β shape variable having two degenerated minima (spherical and deformed) one aims to study for the first time a critical point for a quantum SPT of the first kind [13]. The resulted solution will have at least one free parameter which fixes the height of the barrier which separates the two deformation minima.

In this way one can judge the criticality of a certain nucleus from its potential given by the fits to experimental collective spectrum. It will depend on the position of the ground state energy level relative to the barrier top. The same criterion is valid also for excited states phase transitions which is neglected or simply unattainable in the actual state of the art microscopical calculations. Both directions will extend the critical point assignment to more nuclei, never considered before. Having analytical wave functions allows us to ascertain the equivalence between critical point and shape coexistence phenomena by studying the density distribution of deformation probability for each state. The next step is to apply the same approach to a potential with minima of different depths (**WP 1.2**). In this way one will be able to study the SPT and coexistence phenomena through extended isotopic and isotonic chains and even in entire areas of the nuclide chart. The same program will be further iterated to another separable version of the BH, namely the γ-unstable case (**WP 1.3**). Finally, the central aim of the project is to provide a universal diagonalization basis able to solve the BH with a potential containing higher order quadrupole invariants withs mixed dependence on β and γ variables (**WP 1.4**). Such a model exists only for potentials having a single minimum. It is known as Algebraic Collective Model (ACM) [11,12] and uses basis states corresponding to a modified oscillator. Such a basis is obviously inappropriate for potentials with multiple minima because it cannot be optimised for both minima. Strong collectivity starts to emerge near A=100, while the region of rare-earth nuclei exhibits the greatest variety of collective features. Therefore numerical applications will be performed on medium and heavy nuclei with A>100, with a special attention to the rare earth isotopes where critical behaviour and shape coexistence is mostly expected.

2. Development of analytically solvable models based on parametrized solutions of the BH with energy dependent potentials for a fully geometric description of the shape coexistence in nuclei positioned near proton and neutron closed shells Z, N=50. In such nuclei the presence of states corresponding to single-particle configurations with sizable deformation is a contradicting surprise. Another particularity of these nuclei is the presence of the high phonon energy levels which are not expected to survive and whose collectivity must be reassessed [21]. The evolution of key features pertaining to shape coexistence in such nuclei will be correlated with the values of the adjustable parameters of the potential which ultimately will provide information about the shape. Once again the theoretical model will follow two branches: one associated to γ-unstable conditions of BH (**WP 2.1**) and the other one corresponding to a γ-stable separable version of BH (**WP 2.2**). The PL will employ energy dependent exactly solvable potentials which are extended dynamical group partners of already used harmonic oscillator and Coulomb-like potentials. Other options for the potential will be also explored. The complementary aim of this objective is to explain the failure of the usual phonon description for the low energy spectrum in these nuclei. Another research line to be pursued here is the identification of the DS governing such exotic collective solutions.

C3. Impact

The central challenge of nuclear structure physics is to understand the evolution of structure in atomic nuclei with changing the numbers of their nucleon constituents. Current nuclear structure physics is in the midst of a rebirth due to technological advances in accelerator and instrumentation infrastructure which can presently deliver and measure highly intensive beams of exotic nuclei, as well as continuously increasing computing power put into service of large scale microscopic calculations never before possible. A comprehensive understanding of nuclear structure as a function of N, Z, and A is however still beyond reach. Even if such a goal is achieved, collective models which express structure directly in terms of geometrical degrees of freedom, symmetries, and their associated quantum numbers, provide an invaluable alternative and complementary perspective. In this context, the results stipulated in the project proposal, especially concerning its first objective, will be a large step in the direction of the systematization of deformation related properties of nuclei. Knowing the collective tendencies of nuclei becomes very important as one approaches the neutron drip line, where the deformation increases and the experimental data is scarce. On the other hand, while the energy spectra for the majority of measured nuclei near the stability valley are known up to high spin, the life times and transition probabilities associated to excited states are largely unknown and difficult to measure. The theoretical study of collective excited states is also very important for the γ-ray spectroscopy, which is one of the most powerful methods to study nuclear structure because the de-excitations of these states goes predominantly via γ emission. The theoretical collective model proposed in this project, can offer information on the γ-ray transition energy between two excited levels, the lifetimes of excited states, the multipolarity of the transitions and their electric or magnetic character. This information helps the experimental design and aids the precise measurement of the emitted γ rays. Such theoretical studies have also a notable impact on the Evaluated Nuclear Structure Data File initiative, by providing reference guidelines for the evaluation of the collective excitations. The precise assignment of parity and angular momentum to experimental collective states is in turn essential for the study of reactions and decays involving coupled channels or nuclear isomers. Besides the practical importance of describing the collective features of many distinct nuclear systems, the project’s topic will provide also a solid theoretical output of general scholarly character. Indeed, the developed mathematical, numerical, and computational methods are of general character, being easily transposable to other domains of science such as atomic, molecular, solid state, high energy and mathematical physics. Moreover, the notion of quantum phase transition is specific not only to nuclear physics but many other fields. The topic is very actual, being intensively studied in many important scientific centers and generating dedicated conferences and workshops with a highly multidisciplinary character. On the other hand, despite the empirical observations, the idea of non-local interaction is unfortunately poorly employed in nuclear physics, with only few notable applications to quark systems. Therefore, the PL's objective to explore this opportunity opens a new chapter on the subject of solvable analytical solutions of BH. This in turn will generate completely new collective features whose experimental realization and phenomenological interpretation will be the subject of many works in future. The social impact is reflected in the formation of the scientific and economical independence of the PL and its young collaborators. This is especially important due to the aging of the theoretical physics community and consistent lack of interest from students in fundamental science. Not less important is the increase of the national and especially international visibility of the team members as independent researchers. This will expedite the promotion of the project members to higher academic degrees and positions. One can also add that the cutting edge character of the project's research program will definitely heighten the national scientific status among leading nations.

C4. Methodology

Models, analytical and computational methods used for the project implementation: The theoretical model to be extensively used is the BMM [3,4] in its particular version where the components of the inertial mass tensor are constants. Other models to be referred include spectral generating algebras and IBM. Even in its simplified form, BMM presents a great complexity and variety of solutions depending on the specific form of the potential energy which is directly related to the shape of the nucleus. The most efficient way to describe shape transitions, critical points and shape coexistence phenomena within this model is by using a sextic potential which allows single or double spherical and deformed minima. Due to the factorization of the shape phase space, one can limit ourselves to a sextic oscillator only for the β shape variable, with the γ-angular equation treated within SU(3) and SO(5) spectral generating algebras for γ-stable and respectively for γ-unstable conditions. To solve the eigensystem for the β variable one will use a diagonalization basis constructed from a Fourier-Bessel expansion [22]. The basis will depend on a boundary parameter which in general case must be optimised for each state such that to minimize the dimension of the basis needed for a certain precision. Setting a minimal target precision one can find an optimal value of this boundary parameter which will assure the convergence of the diagonalization for all relevant states and having the same dimension of the basis. The matrix elements within such a basis are expressed in terms of integrals over Bessel functions. These integrals cannot be solved analytically as in the case of Laguerre polynomials used in ACM [11,12] and therefore their calculation is very costly in terms of computer power. This inconvenience will be eliminated in this project by using some recurrence relations in order to discard the numerical integration. The numerical applications will be performed mostly with Fortran codes, whose intermediate results will be confronted with similar calculations made within Mathematica or Maple. The testing of the numerical results will be made on special cases when parameters of the sextic potential allow quasi-exact solutions [23,24,25]. The three parameters defining the sextic potential reduces to two when certain scaling properties of the Schrodinger equation are called upon. The program will be applied in the first stages to sextic potentials with double degenerate minima corresponding to critical point phenomena and shape coexistence. This additional condition leaves a single free parameter, such that one can have a more insightful analysis of the model in what concerns the emergence of criticality or shape coexistence as function of the barrier's height separating the spherical and deformed minima. Such an analysis will be performed on the composition of the wave function and the corresponding density probability distribution of β deformation as well as the distribution of low lying energy states relative to the barrier. The fits to experimental data will also be more affordable with a single adjustable parameter for β excitations. The next stage will be to consider minima in the sextic potential with different depths. The application of this generalized approach in γ-stable and γ-unstable cases will allow a consistent description of SPT throughout large regions of nuclei and the identification of new critical point nuclei. Finally, these parallel descriptions would be unified in a single versatile approach capable to produce collective spectra for double minima potentials in both shape variables γ and β. In what concerns the other major objective, the effort is more analytical than computational. The basic idea is to use a parametrized energy dependent potential to treat nuclei exhibiting a special kind of shape coexistence. The scope of such a solution is to have a generalization of the asymptotic models of energy dependent potentials [19] which was shown to have features compatible with shape coexistence. This will allow to study the evolution of monopole transitions in the candidate nuclei as function of the potential's parameters.

Detailed work plan: The well development of the project will be assured by regular project team meetings and several necessary stages: 1) Documentation on the up to date status of the subject. 2) Identification of the key aspects to be followed and original results to be achieved. 3) Delegation of tasks to each member and establishing the chain of collaboration. 4) Cross checking the input of each member to the analytical formalism. 5) Writing new computer codes and adapting the old ones to the new calculations. 6) Devising tests for the numerical output and performing numerical calculations. 7) Gathering recent relevant experimental data, and confronting them with the theoretical predictions. 8) Performing numerical calculations with other models in order to judge the efficiency of the developed theory. 9) Editing scientific papers and subsequent submitting of the preprints to arxhiv.org and other scientific repositories. 10) Dissemination of results by sending private copies to field experts and interested peers, presenting results at international scientific meetings and updating the dedicated web page of the project. The evolution of the project will commence as follows:

Objective 1: All members will be engaged. PL will ascertain the current status of the studied problem and indicate the literature to be consulted by each member. The analytical formalism will be managed mainly by the PL and the other young researcher (YR) with a gradual implication of the postdoc (PD) member. The implementation of the theoretical method into computer codes will be realized by the PL, while the testing will be performed by the other two members. The YR and PD will gather experimental data and run the codes, search for favorable experimental candidates of the theoretical model through numerical fits and compare the final theoretical results with experimental data. The writing of the papers will be the PL's attribute, with specific input of the other members regarding results visualizations through tables and graphics. The program will be closely reiterated four times corresponding to each of the subsequent WP indicated for this objective. The realization of this objective will be extended on the whole period of the project with 6, 5, 5 and 8 months allocated for each intermediate WP.

Objective 2: Implication - PL and PD. The analytical aspect of the issue comprises the major part of the objective and will be treated in a close collaboration between PL and PD. PD will perform numerical calculations under the supervision of the PL. The manuscript editing will be delegated mainly to the PD. The two work packages of this objective will be realized in intervals of 6 months at the start and the end of the project's time period.

Risk assessment: The past results of the team members and the recent publication record of the PL, guarantee the feasibility of the project. Major impediments are not expected. The only concern is the possibility of long running times of the developed codes, which could be efficiently improved by code optimizations, alternative use of different available computing software for full calculations or intermediate steps, and finally by upgrading the hardware.

Deliverables: **PL undertakes to deliver in total at least 6 papers** - 4 regarding the first objective and 2 in connection to the second one, each corresponding to the WP stipulated in the major objectives. All papers will be published in high impact prestigious nuclear physics or multidisciplinary journals. All members will attend international scientific meetings with the scope of presenting and successfully defend the results of the project in front of experts. The scientific output of the project will be thus supplemented with at least the same number of proceedings papers.

C5. Ethical aspects (if appropriate)

The authorship of the resulted papers will be limited to those who have made a significant contribution to their conception, design, execution, or interpretation. PL will ensure that the obtained results are entirely original, and if the work of others have been used, it has been appropriately cited or quoted.

C6. Resources and budget

The most valuable resource of the project is the human one. Therefore the main part of the budget is used for salaries of the personnel of the team. As already existent infrastructure, we mention that all members have a high-end laptop equipped with software suits necessary for: symbolic calculus, Fortran and C++ programming, text editing, data analyzing and graphing. The available specialized software licenses are: 1 Maple, 2 Mathematica, and 2 Origin. There is also a powerful desktop computer for more demanding calculations. The existing office infrastructure in the host institution (printers, scanners, photo copiers, fully equipped conference rooms etc.) supports a good development of all activities stipulated in the project. However, we expect to cover some expenses for office consumption products such as ink toner, paper etc., and to upgrade the hardware and software equipment in order to reduce the calculation times of the developed codes. All members have access to a high speed internet connection through the network of the host institution. This in turn facilitates the electronic access to all major scientific journals and a selection of e-books through the institute subscription. The team also have at their immediate disposal the Physics National Library, which is the largest scientific library in the country. The mobility budget will be used to cover the participation fees, travel expenses, lodging and per diem for attendance of all project members at international scientific meetings. The overhead expenses will be managed by the host institution and will cover a part of administrative staff salaries, rents, paid leaves, equipment maintenance, fees and taxes, utilities, etc. One may assert without any risk of being mistaken, that we have all conditions for performing research at a very high level.

Bugdet Breakdown (in euro):

Budget chapter (expenses)	**2017 (euro)**	**2018 (euro)**	**2019 (euro)**	**Total budget 2017-2019 (euro)**
Personnel	13.891,4	33.775,56	18.739,26	66.406,22
Logistics	0	7.500	0	7.500
Travel	3.500	7.500	3.000	14.000
Indirect	2.608,6	6.224,44	3.260,74	12.093,78
Total	20.000	55.000	25.000	100.000

Budget Breakdown (in lei):

Budget chapter (expenses)	**2017 (lei)**	**2018 (lei)**	**2019 (lei)**	**Total budget 2017-2019 (lei)**
Personnel	62.511,35	151.990	84.326,65	298.828
Logistics	0	33.750	0	33.750
Travel	15.750	33.750	13.500	63.000
Indirect	11.738,65	28.010	14.673,35	54.422
Total	90.000	247.500	112.500	450.000

C7. Bibliography *(maximum 1 page)*

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